

Calc BC - Vectors "Day 5" homework

$$\textcircled{1} \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3 \cos(3t)}{2e^{2t}}$$

$$\textcircled{2} L = \int_0^{\frac{\pi}{2}} \sqrt{(-3 \cos^2 t \cdot \sin t)^2 + (2 \sin t \cdot \cos t)^2} dt$$

$$\textcircled{3} \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{vertical tangent when } \frac{dx}{dt} = 0$$
$$3t^2 - 2t = 0 \Rightarrow t(3t - 2) = 0 \quad t = 0, t = \frac{2}{3}$$

$$\textcircled{4} v(t) = \left\langle 2t, \frac{2}{2t+3} \right\rangle, \quad a(t) = \left\langle 2, \frac{-4}{(2t+3)^2} \right\rangle$$
$$2 \cdot (2t+3)^{-1}$$

oops!!

$$\textcircled{6} \quad x = e^t + 1 \quad y = 2e^{2t}$$
$$e^t = x - 1$$
$$t = \ln|x - 1|$$
$$y = 2e^{2 \cdot \ln|x - 1|} \Rightarrow 2 \cdot e^{\ln(x-1)^2}$$
$$= 2(x-1)^2$$
$$y = 2x^2 - 4x + 2$$

or $e^t = x - 1$

$$e^{2t} = (x-1)^2$$
$$\text{so } y = 2 \cdot (x-1)^2$$

For some reason, I skipped #5. It will be on the next page. sorry if this caused any trauma.

$$\textcircled{5} \quad x(1) = 1 \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 4}{6t - 4} \quad \left. \frac{dy}{dx} \right|_{t=1} = \frac{-1}{2}$$

$$y(1) = -3$$

$$y + 3 = -\frac{1}{2}(x - 1)$$

$$\textcircled{7} \quad \text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \Big|_{t=2} \Rightarrow \sqrt{(5 \sin(5t))^2 + (3t^2)^2} \Big|_{t=2}$$

$$= 12.304$$

$$\textcircled{8} \quad \text{a) Magnitude} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \Big|_{t=1} \Rightarrow \sqrt{((t-2)^2)^2 + (2t-4)^2} \Big|_{t=1}$$

$$= \sqrt{5}$$

$$\text{b) Distance} = \int_0^1 \sqrt{((t-2)^2)^2 + (2t-4)^2} dt$$

$$= 3.816$$

$$\text{c) } v(t) = 0, \quad v(t) = \langle (t-2)^2, 2t-4 \rangle = \langle 0, 0 \rangle \text{ when } t=2$$

$$s(2) = (4, 0)$$

$$\textcircled{9} \quad \text{Speed} = \sqrt{(1 + \tan(t^2))^2 + (3e^{\sqrt{t}})^2} \Big|_{t=5} = 28.083$$

$$a(t) = \left\langle 2t \sec^2(t^2), \frac{3e^{\sqrt{t}}}{2\sqrt{t}} \right\rangle$$

$$a(5) = \langle 10.178, 6.277 \rangle$$

$$\textcircled{10} \quad 3t + 2 \sin t = 5 \text{ when } t = 1.079$$

$$v(t) = \langle 1 - \sin t, 3 + 2 \cos t \rangle$$

$$v(1.079) = \langle 0.119, 3.944 \rangle$$